ENGINEERING MECHANICS
STATICS

J. L. MERIAM
L. G. KRAIGE
J. N. BOLTON

9th Edition

WILEY

Engineering Mechanics

Volume 1

Statics

Ninth Edition

Engineering Mechanics

Volume 1

Statics

Ninth Edition

J.L. MERIAM

L.G. KRAIGE
Virginia Polytechnic Institute and State University

J.N. BOLTON
Bluefield State College

Wiley

On the cover: The Auditorio de Tenerife "Adán Martin" is located in Santa Cruz de Tenerife, the capital of the Canary Islands, Spain. It was designed by architect Santiago Calatrava Valls and was opened in 2003.

This series of textbooks was begun in 1951 by the late Dr. James L. Meriam. At that time, the books represented a revolutionary transformation in undergraduate mechanics education. They became the definitive textbooks for the decades that followed as well as models for other engineering mechanics texts that have subsequently appeared. Published under slightly different titles prior to the 1978 First Editions, this textbook series has always been characterized by logical organization, clear and rigorous presentation of the theory, instructive sample problems, and a rich collection of real-life problems, all with a high standard of illustration. In addition to the U.S. versions, the books have appeared in SI versions and have been translated into many foreign languages. These textbooks collectively represent an international standard for undergraduate texts in mechanics.

The innovations and contributions of Dr. Meriam (1917–2000) to the field of engineering mechanics cannot be overstated. He was one of the premier engineering educators of the second half of the twentieth century. Dr. Meriam earned the B.E., M.Eng., and Ph.D. degrees from Yale University. He had early industrial experience with Pratt and Whitney Aircraft and the General Electric Company. During the Second World War he served in the U.S. Coast Guard. He was a member of the faculty of the University of California—Berkeley, Dean of Engineering at Duke University, a faculty member at the California Polytechnic State University, and visiting professor at the University of California—Santa Barbara, finally retiring in 1990. Professor Meriam always placed great emphasis on teaching, and this trait was recognized by his students wherever he taught. He was the recipient of several teaching awards, including the Benjamin Garver Lamme Award, which is the highest annual national award of the American Society of Engineering Education (ASEE).

Dr. L. Glenn Kraige, coauthor of the *Engineering Mechanics* series since the early 1980s, has also made significant contributions to mechanics education. Dr. Kraige earned his B.S., M.S., and Ph.D. degrees at the University of Virginia, principally in aerospace engineering, and he is Professor Emeritus of Engineering Science and Mechanics at Virginia Polytechnic Institute and State University. During the mid-1970s, I had the singular pleasure of chairing

Professor Kraige's graduate committee and take particular pride in the fact that he was the first of my fifty-four Ph.D. graduates. Professor Kraige was invited by Professor Meriam to team with him, thereby ensuring that the Meriam legacy of textbook authorship excellence would be carried forward to future generations of engineers.

In addition to his widely recognized research and publications in the field of spacecraft dynamics, Professor Kraige has devoted his attention to the teaching of mechanics at both introductory and advanced levels. His outstanding teaching has been widely recognized and has earned him teaching awards at the departmental, college, university, state, regional, and national levels. These awards include the Outstanding Educator Award from the State Council of Higher Education for the Commonwealth of Virginia. In 1996, the Mechanics Division of ASEE bestowed upon him the Archie Higdon Distinguished Educator Award. The Carnegie Foundation for the Advancement of Teaching and the Council for Advancement and Support of Education awarded him the distinction of Virginia Professor of the Year for 1997. In his teaching, Professor Kraige stresses the development of analytical capabilities along with the strengthening of physical insight and engineering judgment. Since the early 1980s, he has worked on personal-computer software designed to enhance the teaching/learning process in statics, dynamics, strength of materials, and higher-level areas of dynamics and vibrations.

Continuing as coauthor for this edition is Dr. Jeffrey N. Bolton, Associate Professor of Mechanical Engineering Technology and Director of Digital Learning at Bluefield State College. Dr. Bolton earned his B.S., M.S., and Ph.D. in Engineering Mechanics from Virginia Polytechnic Institute and State University. His research interests include automatic balancing of six-degree-of-freedom elastically-mounted rotors. He has a wealth of teaching experience, including at Virginia Tech, where he was the 2010 recipient of the Sporn Teaching Award for Engineering Subjects, which is primarily chosen by students. In 2014, Professor Bolton received the Outstanding Faculty Award from Bluefield State College. Professor Bolton was selected as the 2016 West Virginia Professor of the Year by the Faculty Merit Foundation. He has the unusual ability to set

high levels of rigor and achievement in the classroom while establishing a high degree of rapport with his students. In addition to maintaining time-tested traditions for future generations of students, Dr. Bolton brings effective application of technology to this textbook series.

The Ninth Edition of *Engineering Mechanics* continues the same high standards set by previous editions and adds new features of help and interest to students. It contains a vast collection of interesting and instructive problems. The faculty and students privileged to teach or study from the Meriam/Kraige/Bolton *Engineering Mechanics* series will benefit from several decades of investment by three highly

accomplished educators. Following the pattern of the previous editions, this textbook stresses the application of theory to actual engineering situations, and at this important task it remains the best.

JOHN L. JUNKINS
Distinguished Professor of Aerospace Engineering
Holder of the Royce E. Wisebaker '39 Chair in
Engineering Innovation
Texas A&M University
College Station, Texas

Preface

Engineering mechanics is both a foundation and a framework for most of the branches of engineering. Many of the topics in such areas as civil, mechanical, aerospace, and agricultural engineering, and of course engineering mechanics itself, are based upon the subjects of statics and dynamics. Even in a discipline such as electrical engineering, practitioners, in the course of considering the electrical components of a robotic device or a manufacturing process, may find themselves first having to deal with the mechanics involved.

Thus, the engineering mechanics sequence is critical to the engineering curriculum. Not only is this sequence needed in itself, but courses in engineering mechanics also serve to solidify the student's understanding of other important subjects, including applied mathematics, physics, and graphics. In addition, these courses serve as excellent settings in which to strengthen problem-solving abilities.

Philosophy

The primary purpose of the study of engineering mechanics is to develop the capacity to predict the effects of force and motion while carrying out the creative design functions of engineering. This capacity requires more than a mere knowledge of the physical and mathematical principles of mechanics; also required is the ability to visualize physical configurations in terms of real materials, actual constraints, and the practical limitations which govern the behavior of machines and structures. One of the primary objectives in a mechanics course is to help the student develop this ability to visualize, which is so vital to problem formulation. Indeed, the construction of a meaningful mathematical model is often a more important experience than its solution. Maximum progress is made when the principles and their limitations are learned together within the context of engineering application.

There is a frequent tendency in the presentation of mechanics to use problems mainly as a vehicle to illustrate theory rather than to develop theory for the purpose of solving problems. When the first view is allowed to predominate, problems tend to become overly idealized and unrelated to engineering with the result that the exercise becomes dull, academic, and uninteresting. This approach deprives the student of valuable experience in formulating problems and thus of discovering the need for and meaning of theory. The second view provides by far the stronger motive for learning theory and leads to a better balance between theory and application. The crucial role played by interest and purpose in providing the strongest possible motive for learning cannot be overemphasized.

Furthermore, as mechanics educators, we should stress the understanding that, at best, theory can only approximate the real world of mechanics rather than the view that the real world approximates the theory. This difference in philosophy is indeed basic and distinguishes the *engineering* of mechanics from the *science* of mechanics.

Over the past several decades, several unfortunate tendencies have occurred in engineering education. First, emphasis on the geometric and physical meanings of prerequisite mathematics appears to have diminished. Second, there has been a significant reduction and even elimination of instruction in graphics, which in the past enhanced the visualization and representation of mechanics problems. Third, in advancing the mathematical level of our treatment of mechanics, there has been a tendency to allow the notational manipulation of vector operations to mask or replace geometric visualization. Mechanics is inherently a subject which depends on geometric and physical perception, and we should increase our efforts to develop this ability.

A special note on the use of computers is in order. The experience of formulating problems, where reason and judgment are developed, is vastly more important for the student than is the manipulative exercise in carrying out the solution. For this reason, computer usage must be carefully controlled. At present, constructing free-body diagrams and formulating governing equations are best done with pencil and paper. On the other hand, there are instances in which the *solution* to the governing equations can best be carried out and displayed using the computer. Computer-oriented problems should be genuine in the sense that there is a condition of design or criticality to be found, rather than "makework" problems in which some parameter is varied for no apparent reason other than to force artificial use of the computer. These thoughts have been kept in mind during the design of the computer-oriented problems in the

Ninth Edition. To conserve adequate time for problem formulation, it is suggested that the student be assigned only a limited number of the computer-oriented problems.

As with previous editions, this Ninth Edition of *Engineering Mechanics* is written with the foregoing philosophy in mind. It is intended primarily for the first engineering course in mechanics, generally taught in the second year of study. *Engineering Mechanics* is written in a style which is both concise and friendly. The major emphasis is on basic principles and methods rather than on a multitude of special cases. Strong effort has been made to show both the cohesiveness of the relatively few fundamental ideas and the great variety of problems which these few ideas will solve.

Organization

In Chapter 1, the fundamental concepts necessary for the study of mechanics are established.

In Chapter 2, the properties of forces, moments, couples, and resultants are developed so that the student may proceed directly to the equilibrium of nonconcurrent force systems in Chapter 3 without unnecessarily belaboring the relatively trivial problem of the equilibrium of concurrent forces acting on a particle.

In both Chapters 2 and 3, analysis of two-dimensional problems is presented in Section A before three-dimensional problems are treated in Section B. With this arrangement, the instructor may cover all of Chapter 2 before beginning Chapter 3 on equilibrium, or the instructor may cover the two chapters in the order 2A, 3A, 2B, 3B. The latter order treats force systems and equilibrium in two dimensions and then treats these topics in three dimensions.

Application of equilibrium principles to simple trusses and to frames and machines is presented in Chapter 4 with primary attention given to two-dimensional systems. A sufficient number of three-dimensional examples are included, however, to enable students to exercise more general vector tools of analysis.

The concepts and categories of distributed forces are introduced at the beginning of Chapter 5, with the balance of the chapter divided into two main sections. Section A treats centroids and mass centers; detailed examples are presented to help students master early applications of calculus to physical and geometrical problems. Section B includes the special topics of beams, flexible cables, and fluid forces, which may be omitted without loss of continuity of basic concepts.

Chapter 6 on friction is divided into Section A on the phenomenon of dry friction and Section B on selected machine applications. Although Section B may be omitted if time is limited, this material does provide a valuable experience for the student in dealing with both concentrated and distributed friction forces.

Chapter 7 presents a consolidated introduction to virtual work with applications limited to single-degree-of-freedom systems. Special emphasis is placed on the advantage of the virtual-work and energy method for interconnected systems and stability determination. Virtual work provides an excellent opportunity to convince the student of the power of mathematical analysis in mechanics.

Moments and products of inertia of areas are presented in Appendix A. This topic helps to bridge the subjects of statics and solid mechanics. Appendix C contains a summary review of selected topics of elementary mathematics as well as several numerical techniques which the student should be prepared to use in computer-solved problems. Useful tables of physical constants, centroids, moments of inertia, and conversion factors are contained in Appendix D.

Pedagogical Features

The basic structure of this textbook consists of an article which rigorously treats the particular subject matter at hand, followed by one or more sample problems. For the Ninth Edition, all homework problems have been moved to a special Student Problems section found after Appendix D near the end of the textbook. There is a Chapter Review at the end of each chapter which summarizes the main points in that chapter, and a corresponding Chapter Review Problem set found in the Student Problems section.

Problems

The 89 Sample Problems appear on specially designed pages by themselves. The solutions to typical statics problems are presented in detail. In addition, explanatory and cautionary notes (Helpful Hints) are number-keyed to the main presentation.

There are 898 homework exercises. The problem sets are divided into *Introductory Problems* and *Representative Problems*. The first section consists of simple, uncomplicated problems designed to help students gain confidence with the new topic, while most of the problems in the second section are of average difficulty and length. The problems are generally arranged in order of increasing difficulty. More difficult exercises

appear near the end of the *Representative Problems* and are marked with the triangular symbol ▶. *Computer-Oriented Problems*, marked with an asterisk, appear throughout the problems and also in a special section at the conclusion of the Chapter Review Problems. Problems marked with the student-solution icon **SS** have solutions available onWileyPlus. The answers to all problems have been provided in a special section near the end of the textbook.

In recognition of the need for emphasis on SI units, there are approximately two problems in SI units for every one in U.S. customary units. This apportionment between the two sets of units permits anywhere from a 50–50 emphasis to a 100-percent SI treatment.

A notable feature of the Ninth Edition, as with all previous editions, is the wealth of interesting and important problems which apply to engineering design. Whether directly identified as such or not, virtually all of the problems deal with principles and procedures inherent in the design and analysis of engineering structures and mechanical systems.

Illustrations

In order to bring the greatest possible degree of realism and clarity to the illustrations, the electronic version of this textbook series continues to be produced in full color. It is important to note that color is used consistently for the identification of certain quantities:

- *red* for forces and moments
- *green* for velocity and acceleration arrows
- *orange dashes* for selected trajectories of moving points

Subdued colors are used for those parts of an illustration which are not central to the problem at hand. Whenever possible, mechanisms or objects which commonly have a certain color will be portrayed in that color. All of the fundamental elements of technical illustration which have been an essential part of this *Engineering Mechanics* series of textbooks have been retained. The authors wish to restate the conviction that a high standard of illustration is critical to any written work in the field of mechanics.

Special Features

We have retained the following hallmark features of previous editions:

- All theory portions are constantly reexamined in order to maximize rigor, clarity, readability, and level of friendliness.

- Key Concepts areas within the theory presentation are specially marked and highlighted.
- The Chapter Reviews are highlighted and feature itemized summaries.
- All Sample Problems are printed on specially designed pages for quick identification.
- Within-the-chapter photographs are provided in order to provide additional connection to actual situations in which statics has played a major role.

Resources and Formats

The following items have been prepared to complement this textbook:

Instructor and Student Resources

The following resources are available online at *www.wiley.com/college/meriam*. There may be additional resources not listed.

WileyPlus: A complete online learning system to help prepare and present lectures, assign and manage homework, keep track of student progress, and customize your course content and delivery. Newly added materials for *WileyPlus* include step-by-step video solutions for approximately 200 problems, all of which are similar to those found in the textbook. These author-generated videos illustrate clear and concise solution strategies for students, further emphasizing key concepts and demonstrating sound principles of problem solving in mechanics.

Instructor's Manual: Prepared by the authors and independently checked, fully worked solutions to all problems in the text are available to faculty by contacting their local Wiley representative.

All *figures* in the text are available in electronic format for use in creating lecture presentations.

All *Sample Problems* are available as electronic files for display and discussion in the classroom.

Formats

This Ninth Edition is available in a variety of formats, including conventional print, WileyPlus standalone, standalone alone e-text (now with numerous enhancements), and other bundled formats. Please contact a Wiley representative (www.wiley.com/go/whosmyrep) for more information.

Acknowledgments

Contents

CHAPTER **1**

Introduction to Statics

By Duke.of.arcH - www.flickr.com/photos/dukeofarch/Getty Images, Inc.

Structures which support large forces must be designed with the principles of mechanics foremost in mind. In this view of Sydney, Australia, one can see a variety of such structures.

1/1   Mechanics

Mechanics is the physical science which deals with the effects of forces on objects. No other subject plays a greater role in engineering analysis than mechanics. Although the principles of mechanics are few, they have wide application in engineering. The principles of mechanics are central to research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow, electrical machines and apparatus, and molecular, atomic, and subatomic behavior. A thorough understanding of this subject is an essential prerequisite for work in these and many other fields.

Mechanics is the oldest of the physical sciences. The early history of this subject is synonymous with the very beginnings of engineering. The earliest recorded writings in mechanics are those of Archimedes (287–212 B.C.) on the principle of the lever and the principle of buoyancy. Substantial progress came later with the formulation of the laws of vector combination of forces by Stevinus (1548–1620), who also formulated most of the principles of statics. The first investigation of a dynamics problem is credited to Galileo (1564–1642) for his experiments with falling stones. The accurate formulation of the laws of motion, as well as the law of gravitation, was made by Newton (1642–1727), who also conceived the idea of the infinitesimal in mathematical analysis. Substantial
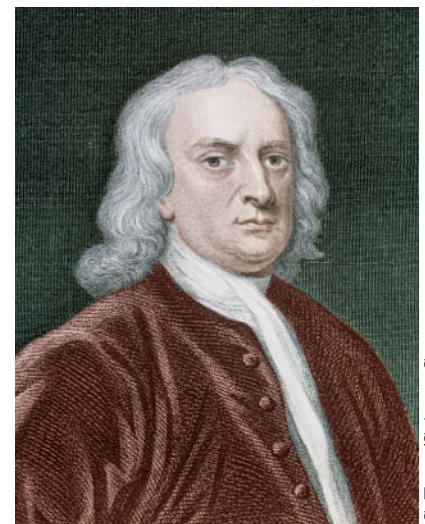
S. Terry/Science Source

Sir Isaac Newton

1

contributions to the development of mechanics were also made by da Vinci, Varignon, Euler, D'Alembert, Lagrange, Laplace, and others.

In this book we will be concerned with both the development of the principles of mechanics and their application. The principles of mechanics as a science are rigorously expressed by mathematics, and thus mathematics plays an important role in the application of these principles to the solution of practical problems.

The subject of mechanics is logically divided into two parts: *statics*, which concerns the equilibrium of bodies under action of forces, and *dynamics*, which concerns the motion of bodies. *Engineering Mechanics* is divided into these two parts, *Vol. 1 Statics* and *Vol. 2 Dynamics*.

1/2   Basic Concepts

The following concepts and definitions are basic to the study of mechanics, and they should be understood at the outset.

Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*. Thus force is a vector quantity, and its properties are discussed in detail in Chapter 2.

A *particle* is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point. We often choose a particle as a differential element of a body. We may treat a body as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it.

Rigid body. A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand. For instance, the calculation of the tension in the cable which supports the boom of a mobile crane under load is essentially unaffected by the small internal deformations in the structural members of the boom. For the purpose, then, of determining the external forces which act on the boom, we may treat it as a rigid body. Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium. Determination of the internal deformations belongs to the study of the mechanics of deformable bodies, which normally follows statics in the curriculum.

1/3   Scalars and Vectors

We use two kinds of quantities in mechanics—scalars and vectors. *Scalar quantities* are those with which only a magnitude is associated. Examples of scalar quantities are time, volume, density, speed, energy, and mass. *Vector quantities*, on the other hand, possess direction as well as magnitude, and must obey the parallelogram

law of addition as described later in this article. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum. Speed is a scalar. It is the magnitude of velocity, which is a vector. Thus velocity is specified by a direction as well as a speed.

Vectors representing physical quantities can be classified as free, sliding, or fixed.

A *free vector* is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector. This vector describes equally well the direction and magnitude of the displacement of every point in the body. Thus, we may represent the displacement of such a body by a free vector.

A *sliding vector* has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole,* and thus it is a sliding vector.

A *fixed vector* is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force. In this instance the forces and deformations within the body depend on the point of application of the force, as well as on its magnitude and line of action.

Conventions for Equations and Diagrams

A vector quantity **V** is represented by a line segment, **Fig. 1/1**, having the direction of the vector and having an arrowhead to indicate the sense. The length of the directed line segment represents to some convenient scale the magnitude $|\mathbf{V}|$ of the vector, which is printed with lightface italic type V. For example, we may choose a scale such that an arrow one inch long represents a force of twenty pounds.

In scalar equations, and frequently on diagrams where only the magnitude of a vector is labeled, the symbol will appear in lightface italic type. Boldface type is used for vector quantities whenever the directional aspect of the vector is a part of its mathematical representation. When writing vector equations, *always* be certain to preserve the mathematical distinction between vectors and scalars. In handwritten work, use a distinguishing mark for each vector quantity, such as an underline, $\underline{V}$, or an arrow over the symbol, $\overrightarrow{V}$, to take the place of boldface type in print.
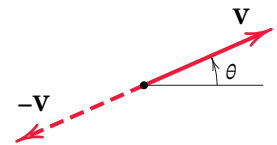
FIGURE 1/1

Working with Vectors

The direction of the vector **V** may be measured by an angle θ from some known reference direction as shown in **Fig. 1/1**. The negative of **V** is a vector **−V** having the same magnitude as **V** but directed in the sense opposite to **V**, as shown in **Fig. 1/1**.

Vectors must obey the parallelogram law of combination. This law states that two vectors **V**$_1$ and **V**$_2$, treated as free vectors, **Fig. 1/2a**, may be replaced by their equivalent vector **V**, which is the diagonal of the parallelogram formed by **V**$_1$ and **V**$_2$ as its two sides, as shown in **Fig. 1/2b**. This combination is called the *vector sum* and is represented by the vector equation

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

*This is the *principle of transmissibility*, which is discussed in Art. 2/2.
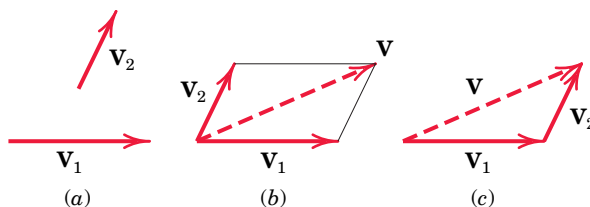
FIGURE 1/2

where the plus sign, when used with the vector quantities (in boldface type), means *vector* and not *scalar* addition. The scalar sum of the magnitudes of the two vectors is written in the usual way as $V_1 + V_2$. The geometry of the parallelogram shows that $V \neq V_1 + V_2$.

The two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, again treated as free vectors, may also be added head-to-tail by the triangle law, as shown in **Fig. 1/2c**, to obtain the identical vector sum $\mathbf{V}$. We see from the diagram that the order of addition of the vectors does not affect their sum, so that $\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1$.

The difference $\mathbf{V}_1 - \mathbf{V}_2$ between the two vectors is easily obtained by adding $-\mathbf{V}_2$ to $\mathbf{V}_1$ as shown in **Fig. 1/3**, where either the triangle or parallelogram procedure may be used. The difference $\mathbf{V}'$ between the two vectors is expressed by the vector equation

$$\mathbf{V}' = \mathbf{V}_1 - \mathbf{V}_2$$

FIGURE 1/3

where the minus sign denotes *vector subtraction*.

Any two or more vectors whose sum equals a certain vector $\mathbf{V}$ are said to be the *components* of that vector. Thus, the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ in **Fig. 1/4a** are the components of $\mathbf{V}$ in the directions 1 and 2, respectively. It is usually most convenient to deal with vector components which are mutually perpendicular; these are called *rectangular components*. The vectors $\mathbf{V}_x$ and $\mathbf{V}_y$ in **Fig. 1/4b** are the x- and y-components, respectively, of $\mathbf{V}$. Likewise, in **Fig. 1/4c**, $\mathbf{V}_{x'}$ and $\mathbf{V}_{y'}$ are the x'- and y'-components of $\mathbf{V}$. When expressed in rectangular components, the direction of the vector with respect to, say, the x-axis is clearly specified by the angle θ, where

$$\theta = \tan^{-1} \frac{V_y}{V_x}$$

A vector $\mathbf{V}$ may be expressed mathematically by multiplying its magnitude V by a vector $\mathbf{n}$ whose magnitude is one and whose direction coincides with that of $\mathbf{V}$. The vector $\mathbf{n}$ is called a *unit vector*. Thus,

$$\mathbf{V} = V\mathbf{n}$$

In this way both the magnitude and direction of the vector are conveniently contained in one mathematical expression. In many problems, particularly three-dimensional
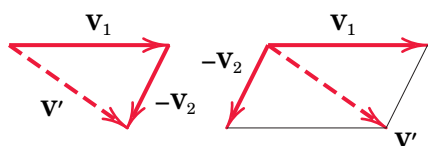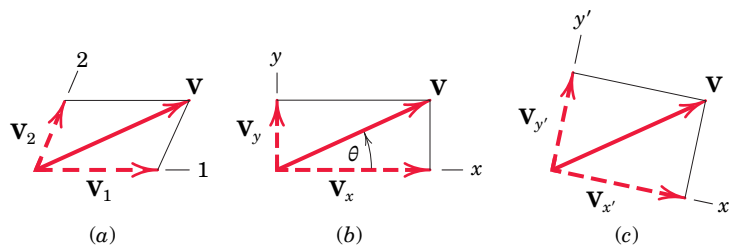

FIGURE 1/4

ones, it is convenient to express the rectangular components of **V**, , in terms of unit vectors **i**, **j**, and **k**, which are vectors in the x-, y-, and z-directions, respectively, with unit magnitudes. Because the vector **V** is the vector sum of the components in the x-, y-, and z-directions, we can express **V** as follows:

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$

We now make use of the *direction cosines l, m*, and n of **V**, which are defined by

$$l = \cos\theta_x \qquad m = \cos\theta_y \qquad n = \cos\theta_z$$

Thus, we may write the magnitudes of the components of **V** as

$$V_x = lV \qquad V_y = mV \qquad V_z = nV$$

where, from the Pythagorean theorem,

$$V^2 = V_x{}^2 + V_y{}^2 + V_z{}^2$$

Note that this relation implies that $l^2 + m^2 + n^2 = 1$.
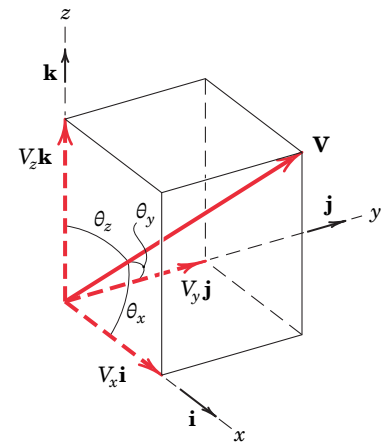
FIGURE 1/5

1/4 Newton's Laws

Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity.* Slightly reworded with modern terminology, these laws are:

Law I. A particle remains at rest or continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it.

Law II. The acceleration of a particle is proportional to the vector sum of forces acting on it and is in the direction of this vector sum.

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and *collinear* (they lie on the same line).

The correctness of these laws has been verified by innumerable accurate physical measurements. Newton's second law forms the basis for most of the analysis in dynamics. As applied to a particle of mass m, it may be stated as

$$\mathbf{F} = m\mathbf{a} \tag{1/1}$$

where **F** is the vector sum of forces acting on the particle and **a** is the resulting acceleration. This equation is a *vector* equation because the direction of **F** must agree with the direction of **a**, and the magnitudes of **F** and $m\mathbf{a}$ must be equal.

Newton's first law contains the principle of the equilibrium of forces, which is the main topic of concern in statics. This law is actually a consequence of the second law, since there is no acceleration when the force is zero, and the particle either is at

---

*Newton's original formulations may be found in the translation of his *Principia* (1687) revised by F. Cajori, University of California Press, 1934.

rest or is moving with a uniform velocity. The first law adds nothing new to the description of motion but is included here because it was part of Newton's classical statements.

The third law is basic to our understanding of force. It states that forces always occur in pairs of equal and opposite forces. Thus, the downward force exerted on the desk by the pencil is accompanied by an upward force of equal magnitude exerted on the pencil by the desk. This principle holds for all forces, variable or constant, regardless of their source, and holds at every instant of time during which the forces are applied. Lack of careful attention to this basic law is the cause of frequent error by the beginner.

In the analysis of bodies under the action of forces, it is absolutely necessary to be clear about which force of each action–reaction pair is being considered. It is necessary first of all to *isolate* the body under consideration and then to consider only the one force of the pair which acts *on* the body in question.

1/5 Units

In mechanics we use four fundamental quantities called *dimensions*. These are length, mass, force, and time. The units used to measure these quantities cannot all be chosen independently because they must be consistent with Newton's second law, Eq. 1/1. Although there are a number of different systems of units, only the two systems most commonly used in science and technology will be used in this text. The four fundamental dimensions and their units and symbols in the two systems are summarized in the following table.

Quantity	Dimensional Symbol	SI Units		U.S. Customary Units	
		Unit	Symbol	Unit	Symbol
Mass	M	kilogram	kg	slug	—
Length	L	meter	m	foot	ft
Time	T	second	s	second	sec
Force	F	newton	N	pound	lb

(SI Units: kilogram, meter, second are Base units; U.S. Customary: foot, second, pound are Base units)

SI Units

The International System of Units, abbreviated SI (from the French, Système International d'Unités), is accepted in the United States and throughout the world, and is a modern version of the metric system. By international agreement, SI units will in time replace other systems. As shown in the table, in SI, the units kilogram (kg) for mass, meter (m) for length, and second (s) for time are selected as the base units, and the newton (N) for force is derived from the preceding three by Eq. 1/1. Thus, force (N) = mass (kg) × acceleration (m/s^2) or

$$N = kg \cdot m/s^2$$

Thus, 1 newton is the force required to give a mass of 1 kg an acceleration of 1 m/s^2.

Consider a body of mass m which is allowed to fall freely near the surface of the earth. With only the force of gravitation acting on the body, it falls with an acceleration g toward the center of the earth. This gravitational force is the *weight W* of the body and is found from Eq. 1/1:

$$W(N) = m\,(kg) \times g\,(m/s^2)$$

U.S. Customary Units

The U.S. customary, or British system of units, also called the foot-pound-second (FPS) system, has been the common system in business and industry in English-speaking countries. Although this system will in time be replaced by SI units, for many more years engineers must be able to work with both SI units and FPS units, and both systems are used freely in *Engineering Mechanics*.

As shown in the table, in the U.S. or FPS system, the units of feet (ft) for length, seconds (sec) for time, and pounds (lb) for force are selected as base units, and the slug for mass is derived from Eq. 1/1. Thus, force (lb) = mass (slugs) × acceleration (ft/sec²), or

$$\text{slug} = \frac{\text{lb-sec}^2}{\text{ft}}$$

Therefore, 1 slug is the mass which is given an acceleration of 1 ft/sec² when acted on by a force of 1 lb. If W is the gravitational force or weight and g is the acceleration due to gravity, Eq. 1/1 gives

$$m \text{ (slugs)} = \frac{W \text{ (lb)}}{g \text{ (ft/sec}^2)}$$

Note that seconds is abbreviated as s in SI units, and as *sec* in FPS units.

In U.S. units the pound is also used on occasion as a unit of mass, especially to specify thermal properties of liquids and gases. When distinction between the two units is necessary, the force unit is frequently written as lbf and the mass unit as lbm. In this book we use almost exclusively the force unit, which is written simply as lb. Other common units of force in the U.S. system are the *kilopound* (kip), which equals 1000 lb, and the *ton*, which equals 2000 lb.

The International System of Units (SI) is termed an *absolute* system because the measurement of the base quantity mass is independent of its environment. On the other hand, the U.S. system (FPS) is termed a *gravitational* system because its base quantity force is defined as the gravitational attraction (weight) acting on a standard mass under specified conditions (sea level and 45° latitude). A standard pound is also the force required to give a one-pound mass an acceleration of 32.1740 ft/sec².

In SI units the kilogram is used *exclusively* as a unit of mass—*never* force. In the MKS (meter, kilogram, second) gravitational system, which has been used for many years in non-English-speaking countries, the kilogram, like the pound, has been used both as a unit of force and as a unit of mass.

Primary Standards

Primary standards for the measurements of mass, length, and time have been established by international agreement and are as follows:

Mass. The kilogram is defined as the mass of a specific platinum–iridium cylinder which is kept at the International Bureau of Weights and Measures near Paris, France. An accurate copy of this cylinder is kept in the United States at the National Institute of Standards and Technology (NIST), formerly the National Bureau of Standards, and serves as the standard of mass for the United States.

Length. The meter, originally defined as one ten-millionth of the distance from the pole to the equator along the meridian through Paris, was later defined as the length of a specific platinum–iridium bar kept at the International Bureau of Weights and Measures. The difficulty of accessing the bar and reproducing accurate measurements prompted the adoption of more accurate

The standard kilogram

Omikron/Science Source

and reproducible standards of length for the meter, which is now defined as the distance traveled by light in a vacuum in (1/299 792 458) second.

Time. The second was originally defined as the fraction 1/(86 400) of the mean solar day. However, irregularities in the earth's rotation led to difficulties with this definition, and a more accurate and reproducible standard has been adopted. The second is now defined as the duration of 9 192 631 770 periods of the radiation of a specific state of the cesium-133 atom.

For most engineering work, and for our purpose in studying mechanics, the accuracy of these standards is considerably beyond our needs. The standard value for gravitational acceleration g is its value at sea level and at a 45° latitude. In the two systems these values are

$$\text{SI units} \qquad g = 9.806\ 65 \text{ m/s}^2$$

$$\text{U.S. units} \qquad g = 32.1740 \text{ ft/sec}^2$$

The approximate values of 9.81 m/s^2 and 32.2 ft/sec^2, respectively, are sufficiently accurate for the vast majority of engineering calculations.

Unit Conversions

A list of the SI units used in mechanics is shown in Table D/5 of Appendix D, along with the numerical conversions between U.S. customary and SI units. Although this table is useful for obtaining a feel for the relative size of SI and U.S. units, in time engineers will find it essential to think directly in terms of SI units without converting from U.S. units. In statics we are primarily concerned with the units of length and force, with mass needed only when we compute gravitational force, as explained in Art. 1/6. Unit conversion is unnecessary for the majority of problems in this textbook.

Figure 1/6 depicts examples of force, mass, and length in the two systems of units, to aid in visualizing their relative magnitudes.

FORCE — 9.81 N (2.20 lbf) — 1 lbf (4.45 N) — 32.2 lbf (143.1 N)

MASS — 1 kg (2.20 lbm) — 1 lbm (0.454 kg) — 1 slug or 32.2 lbm (14.59 kg)
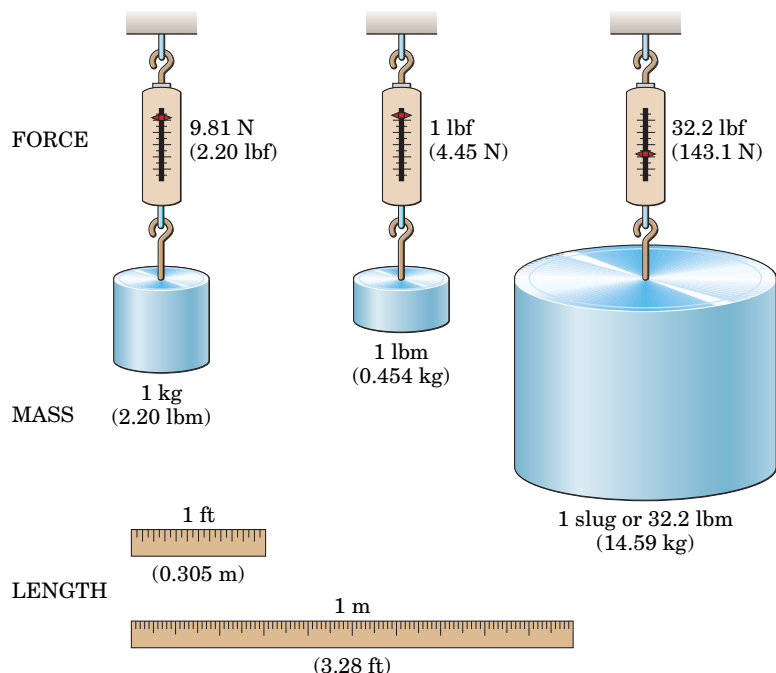
LENGTH — 1 ft (0.305 m) — 1 m (3.28 ft)

FIGURE 1/6

1/6 Law of Gravitation

In statics as well as dynamics we often need to compute the weight of a body, which is the gravitational force acting on it. This computation depends on the *law of gravitation*, which was also formulated by Newton. The law of gravitation is expressed by the equation

$$F = G\,\frac{m_1 m_2}{r^2}$$ (1/2)

where F = the mutual force of attraction between two particles

G = a universal constant known as the *constant of gravitation*

m_1, m_2 = the masses of the two particles

r = the distance between the centers of the particles

The mutual forces F obey the law of action and reaction, since they are equal and opposite and are directed along the line joining the centers of the particles, as shown in **Fig. 1/7**. By experiment the gravitational constant is found to be $G = 6.673(10^{-11})$ m^3/(kg·s^2).

Gravitational Attraction of the Earth

Gravitational forces exist between every pair of bodies. On the surface of the earth the only gravitational force of appreciable magnitude is the force due to the attraction of the earth. For example, each of two iron spheres 100 mm in diameter is attracted to the earth with a gravitational force of 37.1 N, which is its weight. On the other hand, the force of mutual attraction between the spheres if they are just touching is 0.000 000 095 1 N. This force is clearly negligible compared with the earth's attraction of 37.1 N. Consequently the gravitational attraction of the earth is the only gravitational force we need to consider for most engineering applications on the earth's surface.

The gravitational attraction of the earth on a body (its weight) exists whether the body is at rest or in motion. Because this attraction is a force, the weight of a body should be expressed in newtons (N) in SI units and in pounds (lb) in U.S. customary units. Unfortunately, in common practice the mass unit kilogram (kg) has been frequently used as a measure of weight. This usage should disappear in time as SI units become more widely used, because in SI units the kilogram is used exclusively for mass and the newton is used for force, including weight.

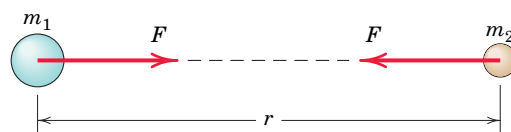The gravitational force which the earth exerts on the moon (foreground) is a key factor in the motion of the moon.

FIGURE 1/7